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A Study of the Graetz Problems in Concentric-Tube Continuous-Contact Countercurrent Separation Processes with Recycles at Both Ends

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Abstract

A mathematical model for the concentric-tube square-off continuous-contact countercurrent separation process with recycles at both ends is developed to predict the concentrations of product. An analytical solution is obtained by means of the orthogonal expansion method. It has been found that only the positive eigenvalues for the enriching section and the negative eigenvalues for the stripping section are required during the calculation of the concentrations of product. Numerical results are presented for the separation of a benzene-*n*-heptane mixture in a concentric-tube thermal diffusion column.

INTRODUCTION

The heat and mass transfer problems commonly encountered in industrial practice occur in the conduit, around the particles, or between phases. The governing equations of the problem are usually simplified under suitable assumptions to obtain an analytical solution. The Graetz problem, in which the fully developed flow with negligible axial diffusion or conduction at steady state is assumed, has been well known for decades. In most cases the eigenfunction may be solved numerically, usually by assuming a power series expansion, by means of a Runge-Kutta integration scheme, or even by using well-known functions such as the confluent hypergeometric function. Many investigators have recently extended the classical Graetz problem with boundary conditions of the first kind to different geometries and to a variety of boundary conditions.

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Typical examples are the consideration of axial conduction or diffusion, the heat generation, and the effects of chemical reaction on heat and mass transfers (*1, 4, 5, 7*). Still, all the velocity distributions concerned did not change sign over the interval in question.

Many separation processes in chemical engineering have been developed in countercurrent operation with internal or external refluxes at both ends. Typical examples are the distillation or extraction in rectifying column, continuous membrane separation, thermal diffusion, countercurrent centrifuges, and mass diffusion. For the equilibrium-stage operation, the McCabe-Thiele, Ponchon-Savarit, or Sorel method is adequate to tackle the problem. For the diffusion-rate operation, there is still lack of a thorough analysis. The difficulties arise not only in the fact that the velocity profile changes sign, but also the concentrations at both ends cannot be specified *a priori*.

For homogeneous mixtures the separation may be reached by the use of external fields, such as pressure and temperature. Therefore, it is the purpose of this work to extend the theory of the Graetz problem in a concentric-tube square-off continuous-contact countercurrent separation process with refluxes at both ends and with uniformly generalized applied fields. We hope that the present formulation will bring a new beginning to a variety of heat or mass transfer problems under the influence of recycling.

COLUMN THEORY

Consider an ideal concentric-tube square-off separation column with the feed introduced at some intermediate position of the column. The column is composed of enriching and stripping sections. The fully developed fluids flow countercurrently with internal or external refluxes at both ends, and the products are withdrawn continuously from both ends. The coordinate is located as shown in Fig. 1, and the lengths of both sections are L_e and L_s, respectively.

The theory is based on the following assumptions:

(1) Physical properties are assumed constant.
(2) Purely laminar flow of the mixture exists in both sections. Therefore, neither radial nor axial mixing may occur.
(3) Ends effects and axial diffusion are negligible.
(4) No bulk flows exist in the radial direction.
(5) External fields, such as temperature and electrical fields, as shown by Crosser et al. (*3*), are applied uniformly in the radial direction to

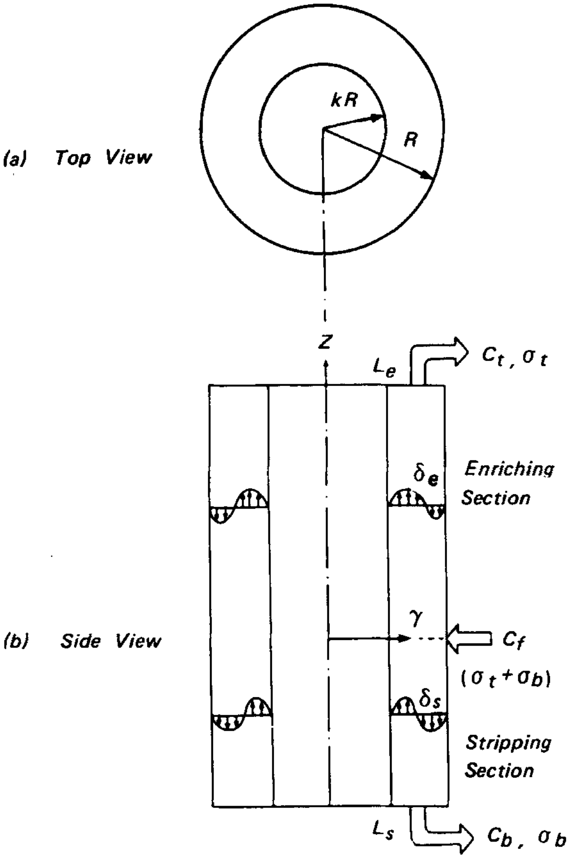

FIG. 1. Schematic diagram of a concentric-tube square-off continuous-contact countercurrent separation process with refluxes at both ends.

cause one component to concentrate at the warm side and at the top of the column.

With this set of assumptions, the mass flux in the radial direction (say, for the enriching section) may be expressed as

$$J_{r,e} = \rho D \left[\frac{\partial C_e}{\partial r} - h(r) \right] \tag{1}$$

in which $h(r)$ is due to the effects of the applied fields. The mass balance for the enriching section is

$$V_e(r)\frac{\partial C_e}{\partial z} = \frac{D}{r}\frac{\partial}{\partial r}\left\{r\frac{\partial C_e}{\partial r} - rh(r)\right\} \tag{2}$$

The boundary conditions for solving Eq. (2) are

$$C_e = C_t, \qquad \text{at } z = L_e \tag{3}$$

$$\partial C_e/\partial r = h(r), \qquad \text{at } r = kR,\ R \tag{4, 5}$$

Equations (4) and (5) can be written because of the existence of impermeable walls. Since it is imperative to have a mixing zone at the end due to reflux, we may impose Eq. (3) on the boundary. Until now, C_t is unspecified.

If the following dimensionless groups are introduced:

$$\zeta = z/L_e,\ \eta = r/R,\ U_e(\eta) = R^2V_e(r)/L_eD,\ f(\eta) = Rh(r) \tag{6}$$

Eqs. (2) to (5) become

$$U_e(\eta)\frac{\partial C_e}{\partial \zeta} = \frac{1}{\eta}\frac{\partial}{\partial \eta}\left\{\eta\frac{\partial C_e}{\partial \eta} - \eta f(\eta)\right\} \tag{7}$$

$$C_e = C_t, \qquad \text{at } \zeta = 1 \tag{8}$$

$$\partial C_e/\partial \eta = f(\eta), \qquad \text{at } \eta = k,\ 1 \tag{9, 10}$$

For the stripping section, all the equations have the same forms as those in the enriching section except that the subscript e and Eq. (8) are replaced by s and Eq. (11), respectively:

$$C_s = C_b, \qquad \text{at } \zeta = -1 \tag{11}$$

Of course, C_b is also not specified at present.

SOLUTION OF CONCENTRATIONS OF PRODUCT

In order to reduce the governing equations to the Sturm-Liouville form, we may assume the concentration profile (say, for the enriching section) to be

$$C_e(\eta,\zeta) = \sum_{m=0}^{\infty} S_{e,m}F_{e,m}(\eta)G_{e,m}(\zeta) + \int_k^{\eta} f(\eta)d\eta \tag{12}$$

where $S_{e,m}$ is the expansion coefficient associated with the eigenvalue $\lambda_{e,m}$. Substituting Eq. (12) into Eqs. (7), (9), and (10) results in

$$G_{e,m}(\zeta) = \exp\left[-\lambda_{e,m}(1-\zeta)\right] \tag{13}$$

$$[\eta F'_{e,m}(\eta)]' - \lambda_{e,m}\eta U_e(\eta)F_{e,m}(\eta) = 0 \tag{14}$$

$$F'_{e,m}(k) = 0 \tag{15}$$

$$F'_{e,m}(1) = 0 \tag{16}$$

where the prime on $F_{e,m}(\eta)$ denotes differentiation with respect to η. The form of Eq. (13) chosen will later be convenient for the derivation of expansion coefficients.

Inspection of Eqs. (14) to (16) shows that they are a special case of the Sturm-Liouville problem for which the velocity profile changes sign over the interval in question. Hence, there may exist a set of real eigenvalues which have the limit points $+\infty$ to $-\infty$. Many methods in the literature have been used to find the eigenvalues and eigenfunctions. For example, one may assume the eigenfunction to be in a power series expansion:

$$F_{e,m}(\eta) = \sum_{n=0}^{\infty} d_{mne}(\eta - 1)^n, \qquad \text{with } d_{moe} = 1 \tag{17}$$

In general, the convergence of the summation in Eq. (17) is slow, especially as the absolute value of the eigenvalue increases. However, this method is still feasible when only few terms of the eigenfunction in the summation of Eq. (12) are needed. Typical examples are separation processes with a large ratio of column length to column thickness. When more eigenfunction terms are needed, they may be expressed in terms of well-known functions, such as the confluent hypergeometric function, to speed the calculation. However, because the power of η in the velocity distribution is more than 2, the merit of such a well-known function is lost. For a generalized velocity distribution, the Runge-Kutta integration scheme may be more appropriate.

For the stripping section, all the equations have the same forms as those in the enriching section except that the subscript e is replaced by s, and Eq. (13) by

$$G_{s,m}(\zeta) = \exp\left[\lambda_{s,m}(1+\zeta)\right] \tag{18}$$

Since the velocity distribution in the stripping section changes sign too, both positive and negative eigenvalues may exist.

It is easy to find the orthogonality conditions for both sections with the weight functions $\eta U_e(\eta)$ and $\eta U_s(\eta)$, respectively. Therefore, applying Eqs. (8), (11), and (12), and integrating from $\eta = k$ to 1, yields

$$\int_k^1 \left[C_t - \int_k^\eta f(\eta) d\eta \right] \eta U_e(\eta) F_{e,m}(\eta) d\eta = S_{e,m} \int_k^1 \eta U_e(\eta) F_{e,m}^2(\eta) d\eta \tag{19}$$

$$\int_k^1 \left[C_b - \int_k^\eta f(\eta) d\eta \right] \eta U_s(\eta) F_{s,m}(\eta) d\eta = S_{s,m} \int_k^1 \eta U_s(\eta) F_{s,m}^2(\eta) d\eta \tag{20}$$

For the case with an eigenvalue of zero in the enriching section, i.e., $\lambda_{e,0} = 0$, we obtain $F_{e,0} = 1 = G_{e,0}$ from Eqs. (14) to (17). Hence, from Eq. (19),

$$S_{e,0} = C_t - \frac{\int_k^1 \left[\int_k^\eta f(\eta) d\eta \right] \eta U_e(\eta) d\eta}{\int_k^1 \eta U_e(\eta) d\eta} \tag{21}$$

Similarly, for the stripping section with $\lambda_{s,0} = 0$,

$$S_{s,0} = C_b - \frac{\int_k^1 \left[\int_k^\eta f(\eta) d\eta \right] \eta U_s(\eta) d\eta}{\int_k^1 \eta U_s(\eta) d\eta} \tag{22}$$

When $\lambda_{e,m} \neq 0$ and $\lambda_{s,m} \neq 0$, Eqs. (19) and (20) can be further reduced to Eqs. (23) and (24) after using standard simplifying techniques.

$$S_{e,m} = \frac{-\int_k^1 \eta F'_{e,m}(\eta) f(\eta) d\eta}{k \lambda_{e,m} F_{e,m}(k) (\partial F'_{e,m}(k) / \partial \lambda_{e,m})} \tag{23}$$

$$S_{s,m} = \frac{-\int_k^1 \eta F'_{s,m}(\eta) f(\eta) d\eta}{k \lambda_{s,m} F_{s,m}(k) (\partial F'_{s,m}(k) / \partial \lambda_{s,m})} \tag{24}$$

It was surprising to find, during the derivation of the above two equations, that the terms associated with C_t and C_b in Eqs. (19) and (20) vanished due to the boundary conditions at the walls. Consequently, the concentration distributions in both sections may be expressed as

$$C_e(\eta,\zeta) = S_{e,0} + \sum_{m=1}^{\infty} S_{e,m}F_{e,m}(\eta)G_{e,m}(\zeta) + \int_k^{\eta} f(\eta)d\eta \tag{25}$$

$$C_s(\eta,\zeta) = S_{s,0} + \sum_{m=1}^{\infty} S_{s,m}F_{s,m}(\eta)G_{s,m}(\zeta) + \int_k^{\eta} f(\eta)d\eta \tag{26}$$

Define the difference of average concentrations for both sections by

$$C_t - C_i = \left\{\int_k^{\delta_e} \eta U_e(\eta)[C_e(\eta,1) - C_e(\eta,0)]d\eta\right\}$$
$$\left\{\frac{\left[\int_k^{\delta_e} \eta U_e(\eta)d\eta\right]^{-1} - \left[\int_{\delta_e}^{1} \eta U_e(\eta)d\eta\right]^{-1}}{2}\right\} \tag{27}$$

$$C_i - C_b = \left\{\int_k^{\delta_s} \eta U_s(\eta)[C_s(\eta,0) - C_s(\eta,-1)]d\eta\right\}$$
$$\left\{\frac{\left[\int_k^{\delta_s} \eta U_s(\eta)d\eta\right]^{-1} - \left[\int_{\delta_s}^{1} \eta U_s(\eta)d\eta\right]^{-1}}{2}\right\} \tag{28}$$

where C_i is the average concentration at the feed position in the column. δ_e and δ_s are the positions where $U_e(\eta)$ and $U_s(\eta)$, respectively, begin to change sign over the interval in question. The reasons why we use the definitions of Eqs. (27) and (28) are explained in the Appendix. Combining the above two equations to eliminate C_i and making use of Eqs. (25) and (26) give the degree of separation for the whole column:

$$\Delta = C_t - C_b = \left\{\sum_{m=1}^{\infty} \frac{S_{e,m}\delta_e F'_{e,m}(\delta_e)[1 - \exp(-\lambda_{e,m})]}{\lambda_{e,m}}\right\}$$

$$\left\{\frac{\left[\int_k^{\delta_e} \eta U_e(\eta)d\eta\right]^{-1} - \left[\int_{\delta_e}^{1} \eta U_e(\eta)d\eta\right]^{-1}}{2}\right\} +$$

$$\left\{\sum_{m=1}^{\infty} \frac{S_{s,m}\delta_s F'_{s,m}(\delta_s)[\exp(\lambda_{s,m}) - 1]}{\lambda_{s,m}}\right\}$$

$$\left\{\frac{\left[\int_k^{\delta_s} \eta U_s(\eta)d\eta\right]^{-1} - \left[\int_{\delta_s}^{1} \eta U_s(\eta)d\eta\right]^{-1}}{2}\right\} \tag{29}$$

A material balance around the whole column gives

$$(\sigma_t + \sigma_b)C_f = \sigma_t C_t + \sigma_b C_b \tag{30}$$

in which σ_t and σ_b are the mass flow rates of product at both ends. Combining Eqs. (29) and (30) results in

$$C_t = C_f + \frac{\Delta}{(1 + \sigma_t/\sigma_b)} \tag{31}$$

$$C_b = C_f - \frac{\Delta(\sigma_t/\sigma_b)}{(1 + \sigma_t/\sigma_b)} \tag{32}$$

Consequently, once all the eigenvalues and associated expansion coefficients are found, we may evaluate C_t and C_b from Eqs. (29), (31), and (32). It is easy to find from Eqs. (29) and (30) that the eigenvalues of zero have no influence on the determination of C_t and C_b. Moreover, in order to make the terms on the right-hand sides of Eqs. (27) and (28) convergent, the negative eigenvalues for $\lambda_{e,m}$ and the positive eigenvalues for $\lambda_{s,m}$ should be omitted.

COMPUTATION ASPECT

For the purpose of illustration, we will consider the separation of a benzene and *n*-heptane mixture in a concentric-tube thermogravitational thermal diffusion column, as shown in Fig. 2. The mass flux due to thermal and ordinary diffusions at any point in the enriching section may be expressed as (*6, 8*)

$$J_{r,e} = \rho D\left[\frac{\partial C_e}{\partial r} - \frac{\alpha C_e(1 - C_e)}{\tilde{T}}\frac{dT}{dr}\right] \tag{33}$$

where α is the thermal diffusion constant and $\tilde{T}$ is defined as the reference temperature of the mixture in the column under batch operation.

We may furthermore assume:

(1) The temperature distribution is determined by conduction in the radial direction only. This is due to the small space between the tube surfaces of the column.

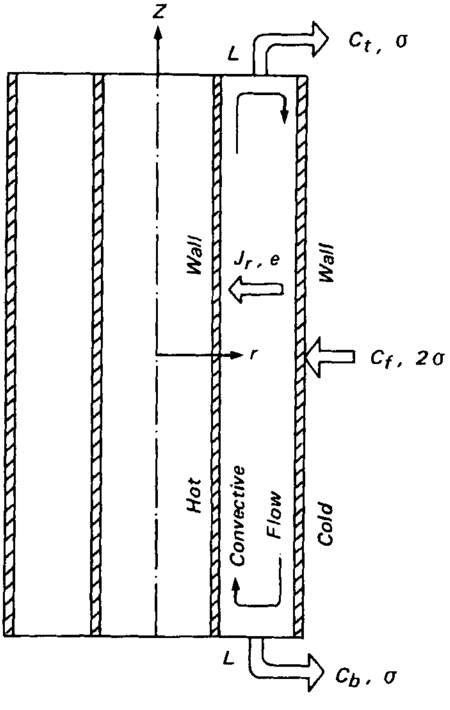

FIG. 2. Schematic diagram of a continuous-flow concentric-tube thermal diffusion column.

(2) The change of concentration in the whole column is in the range of 0.3 to 0.7. Hence, the quadratic form of concentration $C_e(1 - C_e)$ may be regarded as constant and taken approximately as 0.25.

(3) The feed is introduced at the center of the column and the products are withdrawn from both ends at equal flow rates, i.e., $L_e = L_s = L$ and $\sigma_t = \sigma_b = \sigma$.

(4) The convective flows occur in the axial direction only, and the velocity profiles can be obtained from the equations of motion and energy.

Accordingly, we obtain (*12, 14*)

$$f(\eta) = \frac{\alpha C_e(1 - C_e)R}{\tilde{T}} \frac{dT}{dr} \cong \frac{\alpha \Delta T}{4\tilde{T}(\ln k)\eta} \tag{34}$$

$$\tilde{T} = T_1 - \phi - \left(\frac{\Delta T}{\ln k}\right) \tag{35}$$

$$\phi = \frac{-(1 - 4k^2 + 3k^4 - 4k^4 \ln k)(\Delta T)}{4[(1 - k^4)\ln k + (1 - k^2)^2]} \tag{36}$$

$$U_e(\eta) = \frac{R^2 V_e(r)}{LD} = \frac{-\beta g R^4}{LD\mu}\left\{(\phi + \omega_e)\left(\eta^2 - \frac{(k^2 - 1)\ln \eta}{\ln k} - 1\right) - \frac{(\Delta T)}{4 \ln k}(k^2 \ln \eta - \eta^2 \ln k)\right\} \tag{37}$$

$$\omega_e = \frac{2\mu\sigma \ln k}{\rho\pi R^4 \beta g[(1 - k^4)\ln k + (1 - k^2)^2]} \tag{38}$$

We may also obtain δ_e by letting $U_e(\eta)$ equal to zero. For the stripping section, Eqs. (34) to (38) are still valid except that the subscript e is replaced by s, and σ in Eq. (38) is replaced by $-\sigma$. Consequently, Eqs. (23) and (24) reduce to

$$S_{e,m} = \frac{-\alpha(\Delta T)[1 - F_{e,m}(k)]}{4\tilde{T}(\ln k)k\lambda_{e,m}F_{e,m}(k)(\partial F'_{e,m}(k)/\partial \lambda_{e,m})} \tag{39}$$

$$S_{s,m} = \frac{-\alpha(\Delta T)[1 - F_{s,m}(k)]}{4\tilde{T}(\ln k)k\lambda_{s,m}F_{s,m}(k)(\partial F'_{s,m}(k)/\partial \lambda_{s,m})} \tag{40}$$

Let us consider a numerical example for the separation of the benzene-*n*-heptane mixture. Some equipment parameters and physical properties are

$$\begin{aligned}
&R = 1.6\ \text{cm}, L = 60\ \text{cm}, k = 0.96, T_1 = 288.5\ \text{K}, T_2 = 322.5\ \text{K},\\
&\quad \alpha = 1.2,\ g = 980\ \text{cm/s}^2,\ C_f = 0.5,\ \mu = 1.34\\
&\quad \times 10^{-4} \exp(1072/\bar{T})\ \text{g}\,\text{cm}^{-1}\,\text{s}^{-1},\ \beta = 4.314 \times 10^{-2}\\
&\quad \times (550.5 - \bar{T})^{-0.641}\ \text{g}\,\text{cm}^{-3}\,\text{K}^{-1},\ \rho = 5.788(0.181 - 1.635\\
&\quad \times 10^{-4}\,\bar{T})\ \text{g/cm}^3,\ D = [2.47 + 4.65 \times 10^{-2}(\bar{T} - 298)] \times 10^{-5}\ \text{cm}^2/\text{s}
\end{aligned}$$

Using these values, the first two positive eigenvalues for the enriching section and the first two negative eigenvalues for the stripping section have been calculated by Runge-Kutta integration method and are presented in Table 1 with the product rate σ as a parameter. The degree of separation is also evaluated and presented in Table 2 with σ as a parameter. For comparison, we have also calculated the degree of separation from the equation derived by Furry et al. (*6*), i.e.,

$$\Delta_1 = \frac{H}{2\sigma}\left\{1 - \exp\left(\frac{-\sigma L}{K}\right)\right\} \tag{41}$$

in which the curvature effect may be neglected due to the value of k approaching unity ($k = 0.96$). The transport constants in Eq. (41) are defined as

$$H = \frac{2\pi R^4 \alpha\beta\rho g(\Delta T)^2 k(1 - k)^3}{6!\,\mu\bar{T}} \tag{42}$$

$$K = \frac{2\pi R^8 \beta^2 \rho g^2 (\Delta T)^2 k(1 - k)^7}{9!\,D\mu^2} \tag{43}$$

The results are shown in Table 2.

DISCUSSION AND CONCLUSIONS

On the basis of this study, and discussions, some conclusions have been reached.

(1) The equation of separation of the Graetz problem in a concentric-tube square-off continuous-contact countercurrent separation pro-

TABLE

Eigenvalue and Related Expansion Coefficients for

σ (g/min)	m	$\lambda_{e,m}$	$F_{e,m}(k)$	$F'_{e,m}(\delta_e)$	$\dfrac{\partial F'_{e,m}(k)}{\partial \lambda_{e,m}}$
0.5	1	1.76142	$7.95747^{(-1)}$	9.58394	1.08623
	2	$1.71738^{(2)}$	$-9.91557^{(-3)}$	$-9.98764^{(1)}$	$-2.62590^{(2)}$
1.0	1	3.32067	$6.51299^{(-1)}$	$1.64085^{(1)}$	2.05578
	2	$1.91579^{(2)}$	$-5.86416^{(-3)}$	$-1.03415^{(2)}$	$-4.09495^{(2)}$
3.0	1	$1.23783^{(1)}$	$2.25609^{(-1)}$	$3.81979^{(1)}$	8.09614
	2	$3.42734^{(2)}$	$-1.84385^{(-4)}$	$-1.23320^{(2)}$	$-8.44491^{(3)}$
5.0	1	$2.83596^{(1)}$	$5.33745^{(-2)}$	$5.25581^{(1)}$	$2.44430^{(1)}$
	2	$6.71240^{(2)}$	$-5.25163^{(-7)}$	$-1.49387^{(2)}$	$-1.79476^{(6)}$

[a] The numerical values in parentheses represent powers of 10, e.g., $7.95747^{(-1)}$ means 7.95747×10^{-1}.

cess with refluxes at both ends has been derived by using the orthogonal expansion technique. In order to make sure of convergence, only the positive set of eigenvalues for the enriching section and the negative set of eigenvalues for the stripping section are necessary during the calculation of the concentrations of product. This behavior is quite different compared with heat and mass transfer in countercurrent flow without reflux.

(2) For illustration, we have given an example for the separation of a benzene-*n*-heptane mixture in the concentric-tube thermal diffusional column. The first two eigenvalues and associated expansion coefficients for both sections have been calculated by the Runge-Kutta integration method and are presented in Table 1 with the mass flow rate σ as a parameter.

(3) The degree of separation based on the mathematical model formulated here are presented in Table 2 with σ as a parameter. We have also calculated the degree of separation from the equation derived by Furry et al. It is shown that the agreement is quite good for a small mass flow rate. However, as σ increases, some modifications of H and K in Eq. (39) should be made to take into account the effect of the mass flow rate. This behavior is quite evident, since Furry et al. have transformed the partial differential equations, which describe the concentration field, into ordinary differential equations giving the axial concentration gradient. As the mass flow rate increases, the variation of the axial component of the concentration gradient in the radial direction is not small

Both Sections with σ as a Parameter ($C_f = 0.5$)^a

$S_{e,m}$	$\lambda_{s,m}$	$F_{s,m}(k)$	$F'_{s,m}(\delta_s)$	$\dfrac{\partial F'_{s,m}(k)}{\partial \lambda_{s,m}}$	$S_{s,m}$
$1.14366^{(-1)}$	-1.75836	1.25637	$-1.20339^{(1)}$	-1.08504	$-9.11755^{(-2)}$
$1.92535^{(-3)}$	$-1.67516^{(2)}$	$-9.22232^{(1)}$	$-9.05632^{(3)}$	$2.37583^{(2)}$	$2.16521^{(-5)}$
$6.68585^{(-2)}$	-3.55825	1.58280	$-2.74667^{(1)}$	-2.20357	$-4.00329^{(-2)}$
$1.86391^{(-3)}$	$-1.90062^{(2)}$	$-1.67517^{(2)}$	$-1.71325^{(4)}$	$3.92953^{(2)}$	$1.14825^{(-5)}$
$2.91979^{(-2)}$	$-1.22346^{(1)}$	4.38511	$-1.67143^{(2)}$	-8.01413	$-6.71169^{(-3)}$
$1.59767^{(-3)}$	$-3.31797^{(2)}$	$-4.45269^{(3)}$	$-5.40287^{(5)}$	$6.89462^{(3)}$	$3.72736^{(-6)}$
$2.18110^{(-2)}$	$-2.77030^{(1)}$	$1.79794^{(1)}$	$-9.41310^{(2)}$	$-2.36187^{(1)}$	$-1.23041^{(-3)}$
$1.34744^{(-3)}$	$-6.43881^{(2)}$	$-1.28259^{(6)}$	$-1.88501^{(8)}$	$1.20972^{(6)}$	$1.09445^{(-9)}$

enough compared to the variation of the other relevant parameter in radial direction to permit transformation.

(4) Several improved thermal diffusion columns (*2, 8–11, 13–15*) have recently been developed to increase separation efficiency. The present mathematical formulas can also be used in those columns to obtain a more precise prediction of the product concentration.

(5) For the special case that the feed is introduced from the bottom of the column, the stripping section is missing. Hence, only the terms associated with the enriching section are retained. One may also follow the same procedure to obtain C_t, C_b, and Δ, once all the expansion coefficients associated with $\lambda_{e,m}$ are found. For the same reason, if the feed is introduced from the top of the column, one may obtain the solutions once all the expansion coefficients associated with $\lambda_{s,m}$ are found.

TABLE 2

Comparison of the Degree of Separation with That from Eq. (41) with σ as a parameter ($C_f = 0.5$)

σ (g/min)	Δ (%)	Δ_1 (%)	$\left\lvert \dfrac{\Delta - \Delta_1}{\Delta} \right\rvert$ (%)
0.5	16.99	16.99	0.00
1.0	10.34	9.95	3.77
3.0	3.60	3.42	5.00
5.0	2.29	2.05	10.48

(6) During the derivation of this work, some assumptions have been made, such as laminar flow for the mixture, negligible end effects and axial diffusion, etc. Therefore, the influence of the neglected effects, such as turbulence, dependent fluid properties, and a variety of boundary conditions, on the results will be the subject of further studies.

APPENDIX

If the range of integration for Eqs. (27) and (28) is from k to 1 in order to define the difference of concentrations in each section, we will obtain conflicting results, i.e.,

$$C_t - C_i = \frac{\int_k^1 \eta U_e(\eta)[C_e(\eta,1) - C_e(\eta,0)]d\eta}{\int_k^1 \eta U_e(\eta)d\eta}$$

$$= \frac{\sum_{m=1}^{\infty} S_{e,m}[1 - \exp(-\lambda_{e,m})] \int_k^1 \eta U_e(\eta) F_{e,m}(\eta)d\eta}{\int_k^1 \eta U_e(\eta)d\eta}$$

$$= 0 = C_i - C_b \tag{A1}$$

This is due to the fact that velocity distributions change sign in two intervals, k to δ_e (or δ_s) and δ_e (or δ_s) to 1. Consequently, we may define the difference of bulk concentrations in the enriching section by

$$C_t - C_i = \frac{1}{2}\left\{\frac{\int_k^{\delta_e} \eta U_e(\eta)[C_e(\eta,1) - C_e(\eta,0)]d\eta}{\int_k^{\delta_e} \eta U_e(\eta)d\eta}\right.$$

$$\left. + \frac{\int_{\delta_e}^{1} \eta U_e(\eta)[C_e(\eta,1) - C_e(\eta,0)]d\eta}{\int_{\delta_e}^{1} \eta U_e(\eta)d\eta} \right\} \tag{A2}$$

Substituting Eq. (A1) into the above equation results in Eq. (27). For the stripping section, we may follow the same routine to obtain Eq. (28).

SYMBOLS

C	weight fraction of Component 1 in binary mixture
C_f, C_i	C in feed stream and at the feed position of the column, respectively
D	ordinary diffusion coefficient in binary mixtures
d_{mn}	coefficient in the eigenfunction F_m
F_m	eigenfunction associated with eigenvalue λ_m
f	dimensionless function defined in Eq. (6)
G_m	function associated with eigenvalue λ_m in the orthogonal expansion method
g	gravitational acceleration
H	transport constant defined by Eq. (42)
h	function defined in Eq. (1)
J_r	mass flux in the radial direction due to ordinary diffusion and external applied field
K	transport constant defined by Eq. (43)
k	ratio of the outer radius of the inner tube to the inner radius of the outer tube
L	column length of each section
R	inner radius of outer tube
r	coordinate axis in the radial direction
S_m	expansion coefficient associated with eigenvalue λ_m
$\tilde{T}$	reference temperature
T_1, T_2	temperatures of cold wall and hot wall, respectively, in the concentric-tube thermal diffusion column
U	dimensionless velocity defined in Eq. (6)
V	velocity distribution
z	coordinate axis in the axial direction

Greek Letters

α	thermal diffusion constant in binary mixtures
β	defined as $-(\partial\rho/\partial T)$ evaluated at $\tilde{T}$
Δ, Δ_1	degree of separation defined by $C_t - C_b$, and evaluated by Eqs. (29) and (41), respectively
ΔT	defined by $T_2 - T_1$
δ	position at which the velocity distribution changes sign
ζ	dimensionless coordinate defined in Eq. (6)
η	dimensionless coordinate defined in Eq. (6)
λ_m	eigenvalue
μ	viscosity of the mixture
ρ	density of the mixture
σ	mass flow rate of product at each section
ϕ	parameter defined in Eq. (36)
ω	parameter defined in Eq. (38) for the enriching section

Subscripts

b	at the bottom of the column
e	for the enriching section
s	for the stripping section
t	at the top of the column

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